19. Yu. M. Davydov, M. G. Kruglov, A. A. Mednov, and V. A. Nefedov, in: Mathematical Methods of Control and Information Processing [in Russian], Moscow (1982), pp. 48-53.
20. F. Gracco and F. V. Bracco, AIAA J., 21, No. 4, 601-607 (1983).
21. A. Basso, in: Proc. Fourth Task Leaders Conf., Sept. 6-8, 1982, Italy (1983), pp. 41-47.
22. L. De Luca, F. Gamba, et al., in: Proc. Thirty-Fifth Congr. Nazionale Associazione Termotecnica Italiana (AIT), Saint Vincent, Sept. 15-19, 1982, Vol. 2, Italy (1982), pp. 641-654.
23. H. J. Schock and D. I. Sosoka, Numerical Studies of the Formation and Destruction of Vortices in a Motored Four-Stroke Piston—Cylinder Configuration, AIAA Paper No. 497 (1983).
24. P. O. Witze, Sandia Rep. No. 81-8242, Livermore, CA (1981).
25. V. E. Alemasov et al., Thermodynamic and Thermophysical Properties of Combustion Products (Handbook) [in Russian], Vol. 4, Moscow (1973).
26. D. Kwak, W. C. Reynolds, and J. H. Ferziger, in: Report No. TF-5, Stanford Univ., Stanford, CA (1975), p. 189.
27. O. J. McMillan, J. H. Ferziger, and R. S. Rogallo, in: AIAA Paper 80-1339 (1980), pp. 1-10.
28. J. W. Deardorff, J. Fluid Mech., 41, 453-467 (1970).
29. J. W. Deardorff, J. Comput. Phys., 8, 120-133 (1971).
30. J. D. Ramshaw, Informal Report, Los Alamos Scientific Laboratory Report LA-7955-MS (August, 1986).
31. W. C. Rivard, T. D. Butler, and O. A. Farmer, in: Proc. Fourth Int. Conf. Numerical Methods in Fluid Dynamics, Springer-Verlag, New York (1975), pp. 334-340.
32. F. H. Harlow and A. A. Amsden, J. Comput. Phys., 8, 197-213 (1971).
33. C. W. Hirt, in: Numerical Methods in Fluid Mechanics [Russian translation], Moscow (1973), pp. 156-164.
34. B. J. Daly, in: Proc. Fourth Int. Conf. Numerical Methods in Fluid Dynamics, Springer-Verlag, New York (1975).
35. P. O. Witze, in: Stratified Charge Automotive Engines, Inst. Mech. Eng. Conf. Publ., London (1980), pp. 25-31.

SYNTHESIS OF THERMOSTATING DEVICES.

II. MATHEMATICAL MODELS

G. N. Dul'nev and P. A. Korenev

UDC 536.24

The article deals with the procedure of designing (synthesis) of thermostating devices from the position of the systems approach. It compares the possibilities of the mathematical models used with this approach.

In the final analysis the designing of a thermostat consists in the realization of such boundary conditions for the object of thermostating which make it possible to attain the required (specified) space—time temperature field of the object, to ensure the specified error of thermostating and the specified time for attaining the operating regime, to satisfy requirements concerning overall dimensions, power consumption, weight, etc.

From this information on the permissible state of the system (the thermostat and the object of thermostating) we have to find the unknown causal characteristics, i.e., the boundary conditions for the object. These boundary conditions then enable us to choose the design of the thermostat and its operating regime. Such a statement of the problem makes it necessary to solve the inverse problem of heat exchange in a technical system [1]. From among various methods of solving inverse problems we will deal with the method of multiple realization ("scanning") of mathematical models whose essence reduces to the following operations: a) on the basis of some considerations the prototype of the design (basic model)

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 660-667, October, 1986. Original article submitted June 10, 1985.

0022-0841/86/5104-1243$12.50

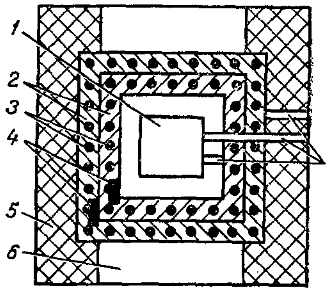

Fig. 1. Diagram showing the design of the thermostating device.

is chosen and the mathematical description of the process under study is constructed; b) with the aid of the chosen mathematical model information on the thermal state of the object of thermostating is ascertained; c) the obtained results are compared with the technical specification (TS), the design is corrected and subsequent calculations are carried out.

This procedure is reminiscent of testing designs by trial and error on actual objects, only these objects are replaced by a mathematical model.

Thus, the described method required the solution of three basic problems, specifically the construction of a set of base models, the construction of a mathematical model for a technical system, and the determination of the most economical method of realizing the scanning procedures. Below we examine possible ways of solving the second problem from among the stated ones in designing thermostating devices.

Modern thermostats (Fig. 1) usually consist of one of several chambers (jackets) 2 inside which the object of thermostating 1 is placed. The jackets contain the control elements 3, in the general case nonuniformly distributed throughout their volume, and temperature sensors 4 which act as sensitive elements of the system of automatic control. The thermal contact of the thermostat with the environment is effected through the thermal insulation 5, the cooling system 6, and through the local connections 7.

In the analysis of the thermal regime of thermostating systems, the characteristic element is the jacket for which mathematical models with various degrees of detailing are used. For instance, in [3] a model was used describing the temperature field of the jacket and of the object with the fullest degree of detailing:

$$c_i(\bar{x}_i,\ t_i)\rho_i(x)\ \frac{\partial t_i}{\partial\tau} = \operatorname{div}(\lambda_i(\bar{x}_i,\ t_i)\operatorname{grad}t_i) + q_v(\bar{x}_i,\ t_i), \tag{1}$$

$$-\lambda_i(\bar{x})\ \frac{\partial t_i}{\partial n} = \alpha_{ij}(\bar{x},\ t_i,\ t_j)(t_i - t_j) + q_s(\bar{x},\ t_i,\ t_j), \tag{2}$$

$$t_i|_{\tau=0} = t_{it}\ \ i = 1,\ 2,\ \ldots,\ N,\ \ \bar{x} = (x_1,\ x_2,\ x_3), \tag{3}$$

where q_v, α_{ij}, t_j, and q_s are functions of the coordinates and temperatures expressing the effect of the interacting elements and of the environment on the jacket under examination. By adequately choosing the form of these functions we can describe the effect of the heater situated inside the jacket.

Equation (1) is a set of three-dimensional nonsteady equations of heat conduction for N structural elements of the thermostat, condition (2) describes the thermal contact of the jackets with each other and with the environment, expressions (3) describe the initial conditions.

The mathematical model (1)-(3) makes it possible to obtain information on the temperature field in any region of a complex system; however, in most cases such information is superfluous, it is difficult to realize and entails much computation time.

The one-dimensional model is simpler, but it is suitable only for obtaining less detailed information on the thermal state of a system [4]:

$$c_i\rho_i\ \frac{\partial t_i}{\partial\tau} = \frac{1}{S_i(r)}\ \frac{\partial}{\partial r}\left[S_i(r)\lambda_i\ \frac{\partial t_i}{\partial r}\right] + q_{vi}(r) + \alpha_{vi}(t_i - u_{vi}), \tag{4}$$

$$\lambda_i \frac{\partial t_i}{\partial r}\bigg|_{r=r_{i,1}} + q_{si,1}(\tau) = \alpha_{i1}(t_i - t_{i-1}),\ i = 2,\ 3,\ \ldots,\ N, \tag{5}$$

$$-\lambda_i \frac{\partial t_i}{\partial r}\bigg|_{r=r_{i,2}} + q_{si,2}(\tau) = \alpha_{i2}(t_i - t_{i+1}),\ i = 1,\ 2,\ \ldots,\ N-1, \tag{6}$$

$$\lambda_1 \frac{\partial t_1}{\partial r}\bigg|_{r=0} = 0,\quad -\lambda_N \frac{\partial t_N}{\partial r}\bigg|_{r=r_{N,2}} + q_{sN,2}(\tau) = \alpha_{N2}(t_N - t_c), \tag{7}$$

$$t_i(r,\ 0) = t_{0i}(r),\ i = 1,\ 2,\ \ldots,\ N. \tag{8}$$

The rules for the transition from the generally accepted system of coordinates to the system of coordinates used in (4)-(8), and also the substantiation of this model are explained in [4].

The system (4)-(8) describes the unidimensional temperature fields of the thermostat jackets. The use of these relations makes it possible to investigate entire classes of objects, and in addition the computer realization of this unidimensional model is much simpler than the realization of (1)-(3).

Even simpler than the two previously mentioned systems is the system of equations with concentrated parameters; it is accepted practice to use it for analyzing the thermal regime of thermostats. In general form this system can be written in the following manner [2, 4, 5]:

$$C_i \frac{dt_{vi}}{d\tau} + t_{si} \sum_{i=1}^{N} \sigma_{ij} - \sum_{j=1}^{N} t_{sj}\sigma_{ij} = P_i, \tag{9}$$

$$t_{vi}\big|_{\tau=0} = t_{vit}. \tag{10}$$

Equation (9) expresses mathematically the law of conservation of energy: the power released in the i-th element is expended on changing the temperature of the i-th volume ($C_i dt_{vi}/d\tau$) and is transmitted from the i-th element to all the other N − 1 elements of the system $\sum_{i=1}^{N} \sigma_{ij}(t_{si} - t_{sj})$. Expression (10) is the initial condition for the i-th element.

The number of unknown temperatures in the system (9)-(10) exceeds the number of equations because the volumetric (v) and the surface (s) temperatures appear in it. Therefore, to close the system, we have to provide additional data establishing a correlation between the sought temperatures. Closure of the system can be effected in several ways [2].

If the internal thermal resistances in the elements of the system are much smaller than the external ones (i.e., $Bi = \alpha_i \ell_i/\lambda_i \ll 1$), it is correct to assume that the temperature field is uniform, i.e.,

$$t_{vi} = t_{si}. \tag{11}$$

In the general case it is necessary to introduce an additional correlation between the unknown mean surface temperatures t_{si} and the mean volumetric temperatures t_{vi}. For that the coefficient of nonuniformity of the temperature field is used:

$$\psi_i = \frac{t_{si} - \tilde{t}_i}{t_{vi} - \tilde{t}_i},\quad \tilde{t}_i = \sum_j \sigma_{ij} t_{sj} / \sum_j \sigma_{ij}. \tag{12}$$

The method of approximately determining ψ_i for bodies with different configurations was presented in [2, 5].

All the other models with whose aid the temperature fields of thermostating systems are analyzed are modifications of the above-mentioned models or their combinations.

Thus, in the synthesis of thermostating systems, several variants of the mathematical models may be used in dependence on the special features of the objects of thermostating and the modifications of the technical specifications. However, the decision which of the models is to be used must be substantiated because there is a danger of either coarsening the investigated process too much or of going needlessly into excessive detail.

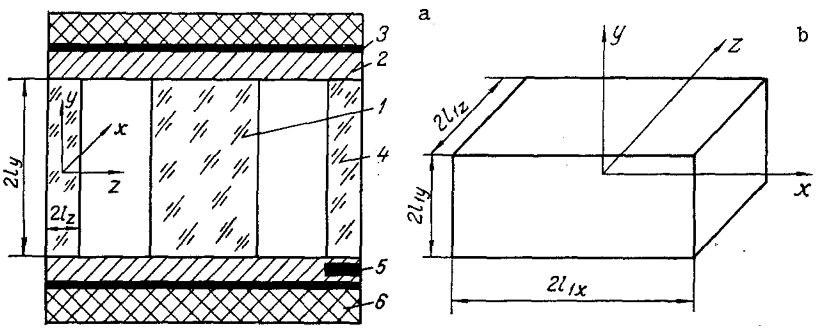

Fig. 2. Diagram showing the design of a thermostat for an optical crystal: a) thermostat; b) crystal.

To attain maximal effectiveness in the choice of the parameters of the thermostat, it is expedient to use the method of stagewise modeling [2, 5] which lately has been widely used. With this method the first stage entails an examination of the system as a whole, with the aid of equations describing the integral interaction between component parts. Then, for a more detailed analysis, some region of the system is chosen, and the entire remaining part is taken into account in the analysis of the thermal regime of the chosen region in the boundary conditions. If the region that is of interest is also a system of bodies, then at the subsequent stage the heat exchange of this new system is examined, etc.

Let us examine the possibility of using the method of stagewise modeling in designing thermostats. At the initial stage we use the mathematical model composed of equations of the type of (4) or (9), which make it possible to determine the mean surface and mean volumetric temperatures. With this model we can determine the basic parameters of the thermostat: the time required for attaining the operating regime, the error of thermostating the mean volumetric or mean surface temperature, the full power of the control element and the regime of its operation, etc. The initial data for calculations by the mentioned models are the ranges of possible change of full thermal capacities C_i, of thermal conductivities σ_{ij}, and also lists of standard elements with their characteristics. The problem of synthesis of the thermostat at this stage consists in choosing from the mentioned ranges such fixed values of the listed parameters which satisfy the requirements of the technical specification for designing the thermostat.

The assumption that it is possible to use a system of equations for the mean surface and mean volumetric temperatures introduces a certain error into the calculation of the quality indicators of the thermostating system. If this error does not exceed the permissible value, we may go over to the following stage, i.e., the analysis of the thermal regime of the most critical elements of the thermostat.

Such an analysis has to be carried out usually only for the object of thermostating. In that case a three-dimensional equation of heat conduction (1) is formulated for the object; this makes it possible to calculate the local temperatures, and from them to go over to the operating parameters of actual objects.

All the data on the thermal state of the remaining part of the investigated system are taken into account in the boundary conditions for the object of thermostating. Thus, the boundary condition (2) at the second stage has the form

$$\lambda_1(\bar{x})\,\frac{\partial t_1}{\partial n} = \bar{\alpha}_{12}(t_1,\ \langle t_{2s}\rangle,\ \bar{x})(t_1 - \langle t_{2s}\rangle) + q_{s1}(\bar{x},\ t_1). \tag{13}$$

The temperature t_j, which is a function of the coordinates, is replaced by the mean surface temperature $\langle t_{2s}\rangle$ nearest to the object of jacket 2, and the local coefficient of heat transfer α_{ij} is replaced by its averaged value $\bar{\alpha}_{12}$. The errors due to such a substitution can be evaluated by the method suggested in [2].

Planning the design of the thermostat at this stage may consist in determining the law of distribution of the power of the control element, the optimization of the conditions of securing the object of thermostating inside the chamber, in the rational disposition of local contacts, etc.

As an example, we will consider the analysis of the thermal regime of a thermostat of an optical crystal. The design of such a thermostat is shown diagrammatically in Fig. 2.

The object of thermostating 1 in the form of a rectangular parallelepiped is situated in chamber 2 of metallic material which is in direct contact with the object through four sides, and the two remaining sides are protected by windows 4 for the passage of a light beam. Situated on the chamber are also the wire heater 3 and the temperature sensor 5. On the outer surface the structure is surrounded by the heat insulation 6.

The problem consists in choosing the parameters of the thermostat design such that they satisfy the requirements of the technical specification regarding the error of thermostating, the time required for attaining the operating regime, and the permissible nonuniformity of the temperature field of the object.

Let us construct the thermal model of the thermostat at the first stage. The evaluation of the temperature gradient across the thickness of the chamber made of metal enables us to regard the chamber as an object with concentrated parameters. The dimensions of the sensor (thermistor) are small compared with the dimensions of the chamber, the processes of heat exchange in the sensor are therefore also described by an equation with concentrated parameters. The heater is taken to be an internal heat source uniformly distributed throughout the volume of the chamber. At this stage of the calculation we also assume that the light windows have a uniform temperature field and are part of the chamber. In addition we assume that the heat insulation has low heat capacity and is characterized solely by the thermal resistance that is part of the thermal resistance between the chamber and the environment.

Thus, the thermal model of the thermostat at the first stage consists of the following elements: the object of thermostating whose temperature field is described by the unidimensional equation of heat conduction, the chamber with uniform temperature distribution, and the sensor. And the mathematical model with two-position control is represented by the system of equations:

$$c_1\rho_1 \frac{\partial t_1}{\partial \tau} = \lambda_1 \frac{1}{S_1(r)} \frac{\partial}{\partial r}\left(S_1(r)\frac{\partial t_1}{\partial r}\right) + q_{v1}, \tag{14}$$

$$C_2 \frac{dt_2}{d\tau} = \sigma_{12}(t_{s1} - t_2) + \sigma_{2c}(t_c - t_2) + P_2, \tag{15}$$

$$C_3 \frac{dt_3}{d\tau} = \sigma_{23}(t_2 - t_3) + \sigma_{3c}(t_c - t_3), \tag{16}$$

$$\left.\frac{\partial t_1}{\partial r}\right|_{r=r_{\mathrm{I},\,2}} = \frac{\sigma_{12}}{\lambda_1 S_{12}}\left(t_1|_{r=r_{\mathrm{I},\,2}} - t_2\right),\quad \lambda_1 \left.\frac{\partial t_1}{\partial r}\right|_{r=0} = 0, \tag{17}$$

$$P_2 = \begin{cases} P_2 \text{ for } t_3 < t_3^{\mathrm{t}} - b \\ \quad\text{or for } \begin{cases} t_3^{\mathrm{t}} - b \leqslant t_3 \leqslant t_3^{\mathrm{t}} + b, \\ \dfrac{dt_3}{d\tau} < 0, \end{cases} \\ 0 \text{ for } t_3 > t_3^{\mathrm{t}} + b \\ \quad\text{or for } \begin{cases} t_3^{\mathrm{t}} - b \leqslant t_3 \leqslant t_3^{\mathrm{t}} + b, \\ \dfrac{dt_3}{d\tau} > 0, \end{cases} \end{cases} \tag{18}$$

$$t_i|_{\tau=0} = t_{i\mathrm{t}}, \tag{19}$$

where t_1, t_2, and t_3 are the temperatures of the object, of the chamber, and of the sensor, respectively; σ_{12}, σ_{23}, σ_{2C}, σ_{3C} are the thermal conductivities between the object and the chamber, the chamber and the sensor, the chamber and the environment, and between the sensor and the environment, respectively: $t_3{}^t$ is the tuning temperature of the sensor: b is the half-width of the zone of non-uniqueness of the regulator.

In this system the equation for the object (14) is a special case of Eq. (4) in which the term taking heat exchange with the inner liquid flowing through is lacking, and the thermal conductivity is not a function of the coordinate. Equations (15) and (16) were obtained from the more general equation (9) for the case when $\psi_1 = \psi_2 = 1$, and the boundary conditions (17) were obtained from (5) for $q_{s1,1} = 0$ and from (7).

With the aid of the numerical realization of the system (14)-(19) we choose the parameters of the design ensuring the required error of thermostating and the time required for attaining the operating regime. The heat capacity C_2 of the chamber, the type and place of attaching the sensor, the power P_2 of the heater, and also the thermal conductivities σ_{12} and σ_{2C} relate to the design parameters.

At the second stage more detailed calculations have to be carried out. This has to do with the fact that the conditions of normal operation of the object can be checked only on the basis of an analysis of the three-dimensional temperature field of the basic elements.

For instance, to determine the nonuniformities of the temperature field of the object caused by the influence of the light windows, we have to determine first the temperature field of the window and choose its thickness such that this influence lies within the limits of the error of thermostating. For that we formulate the steady-state three-dimensional equation of heat conduction for a rectangular parallelepiped without heat sources:

$$\frac{\partial^2 t_0}{\partial x^2} + \frac{\partial^2 t_0}{\partial y^2} + \frac{\partial^2 t_0}{\partial z^2} = 0. \tag{20}$$

The boundary conditions for (20) express the thermal interaction between window, chamber, and object:

$$t_0(x,\ y,\ z)\Big|_{\substack{x=l_x\\ y=l_y}} = t_2, \tag{21}$$

$$\frac{\partial t_0}{\partial x}\Big|_{x=0} = \frac{\partial t_0}{\partial y}\Big|_{y=0} = 0, \tag{22}$$

$$\lambda_0 \frac{\partial t_0}{\partial z}\Big|_{z=-l_z} = \alpha_1 (t_0 - t_c);\quad \lambda_0 \frac{\partial t_0}{\partial z}\Big|_{z=l_z} = \alpha_2 (t_0 - t_{s1}), \tag{23}$$

where t_0 is the temperature of the light window; $2\ell_x$, $2\ell_y$, $2\ell_z$ are the dimensions of the window in the directions of the axes of coordinates; t_2 is the temperature of the chamber; t_C is the ambient temperature; α_1 and α_2 are the heat transfer coefficients from the surface of the window to the environment and toward the side of the crystal.

Condition (21) presupposes equality of the temperatures of the window and of the chamber on the boundaries $x = \ell_x$ and $y = \ell_y$ of their direct contact; condition (22) denotes symmetry of the temperature field relative to the planes $x = 0$ and $y = 0$, and condition (23) describes the heat exchange of the surface of the window with the environment and with the object of thermostating.

In accordance with the method of stagewise modeling, the temperature t_2 of the thermostat chamber in Eqs. (20)-(23) is taken from the solution of the system (14)-(19).

The nonuniformity of the temperature of the object of thermostating is evaluated on the basis of the solution of the three-dimensional nonsteady-state equation of heat conduction which is a linear variant of Eq. (1):

$$c_1\rho_1 \frac{\partial t_1}{\partial \tau} = \lambda_1 \nabla^2 t_1 + q_{v1}. \tag{24}$$

The boundary conditions for (24) express the conditions of thermal interaction of the object with other elements of the thermostat:

$$\frac{\partial t_1(x,\ y,\ z)}{\partial z}\Big|_{z=0} = 0;\ \lambda_1 \frac{\partial t_1}{\partial z}\Big|_{z=l_{1z}} = \alpha_2 (t_1 - t_0), \tag{25a}$$

$$t_1(x,\ y,\ z)\Big|_{\substack{x=l_{1x}\\ y=l_{1y}}} = t_2;\quad \frac{\partial t_1}{\partial x}\Big|_{x=0} = \frac{\partial t_1}{\partial y}\Big|_{y=0} = 0, \tag{25b}$$

$$t_1(x,\ y,\ z,\ \tau)\Big|_{\tau=0} = t_{1t}(x,\ y,\ z), \tag{25c}$$

where ℓ_{1x}, ℓ_{1y}, ℓ_{1z} are the dimensions of the object of thermostating in the directions of the axes of coordinates; t_0, t_2 are the temperatures of the light window and of the chamber, respectively, obtained at the preceding stages after the problems (14)-(19) and (20)-(23) had been solved.

In accordance with (25) the object takes part in symmetric heat exchange with the light windows along the z-axis (25a), and along the other two axes it is in ideal contact with the chamber (25b). Condition (25c) closes the system.

The presented example shows that the described approach makes it possible from the same positions to arrive at the synthesis of the design of thermostats for different objects.

A further task is to work out actual algorithms for choosing the design parameters satisfying the requirements of the technical specification.

NOTATION

t_i, t_j, temperature of the i-th and j-th element, respectively, of the thermostat; t_{vi}, t_{si}, mean volumetric and mean surface temperature, respectively, of the i-th element of the thermostat; C_i, c_i, ρ_i, λ_i, full and specific heat capacity, density, and thermal conductivity, respectively, of the i-th element of the thermostat; q_{vi}, q_{si}, specific power of the volumetric and surface heat sources, respectively; τ, time; $S_i(r)$, running area of the isothermal surface; r, generalized coordinate; σ_{ij}, thermal conductivity between the i-th and j-th elements of the thermostat; α_{vi}, volumetric heat-transfer coefficient with the inner convective medium; α_{N2}, heat-transfer coefficient of the element of the thermostat with the environment; u_{vi}, temperature of the inner convective medium; $q_{si,1}$, $q_{si,2}$, specific power of the surface heat sources on the inner and outer surface, respectively, of the i-th jacket; P_i, full power of the heat sources in the i-th element; i, j, subscripts denoting the ordinal numbers of the elements of the thermostat.

LITERATURE CITED

1. O. M. Alifanov, Inzh.-Fiz. Zh., 33, No. 6, 972-981 (1977).
2. G. N. Dul'nev and A. V. Sigalov, Inzh.-Fiz. Zh., 45, No. 4, 651-656 (1983).
3. G. N. Dul'nev, V. I. Egorov, and V. G. Parfenov, Inzh. Fiz. Zh., 38, No. 6, 1099-1105 (1980).
4. N. A. Yaryshev and L. B. Andreeva, Thermal Calculation of Thermostats [in Russian], Leningrad (1984).
5. G. N. Dul'nev, Heat and Mass Exchange of RÉA [in Russian], Moscow (1984).

PARAMETRIC METHOD FOR THE SOLUTION OF AN ILL-POSED INVERSE HEAT-CONDUCTION PROBLEM IN APPLICATION TO THE OPTIMIZATION OF THERMAL REGIMES

V. M. Vigak and V. L. Fal'kovskii

UDC 517.977.56:536.12

A method is proposed for the stable approximate solution of an ill-posed inverse heat-conduction problem, to which the investigated problem of optimal control of the thermal regime of a rigid body is reduced.

The timeliness of nondestructive testing problems and the difficulties of reconstructing the temperature field and the thermophysical characteristics of an object from experimental results have fostered the rapid development of identification methods in heat conduction [1-3]. It has been shown [4] that a number of problems in the control of the thermal regime of a rigid body can also be solved by reducing them to an inverse heat-conduction problem (IHCP). In the latter case, as a rule, certain characteristic (thermal or thermomechanical)

Institute of Applied Problems of Mechanics and Mathematics, Academy of Sciences of the Ukrainian SSR, Lvov.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 4, pp. 668-673, October, 1986. Original article submitted August 26, 1985.

0022-0841/86/5104-1249$12.50